

## Metastable domains of the landscape

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**ABSTRACT:** We argue that the vast majority of flux vacua with small cosmological constant are unstable to rapid decay to a big crunch. Exceptions are states with large compactification volume and supersymmetric and approximately supersymmetric states. Neither weak string coupling, warping, or the existence of very light particles are, by themselves, enough to render states reasonably metastable. We speculate, as well, about states which might be cosmological attractors.

**KEYWORDS:** dS vacua in string theory, Flux compactifications.

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## 1. Introduction: stability in the landscape

There is now a widely held belief that string theory possesses a vast array of metastable states, the *landscape*. The evidence for the existence of these states is circumstantial but, for many, compelling.<sup>1</sup> The usual strategy is to study a classical effective action (e.g. for IIB theories on Calabi-Yau spaces) and to look for classically stable stationary points. If at the stationary point, the system has a small coupling and large internal volume, this is strongly suggestive that the state is sensible and metastable.

Much of the focus of landscape studies has been on states with some degree of supersymmetry. These are easier to study as supersymmetry provides an added degree of theoretical control. KKLT [1] exhibited states in which it appears that all moduli are fixed in a regime of weak coupling and large compactification radius. They (and subsequently others [2]), argued that a substantial number of such states would exhibit dynamical supersymmetry breaking with positive cosmological constant. More generally, if one considers likely mechanisms for supersymmetry breaking among the known supersymmetric states, it is likely that a finite fraction have hierarchically small breaking scales, as expected from conventional ideas about naturalness [3].

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<sup>1</sup>Notable nay sayers include Tom Banks and others less vocal.

While some degree of calculational control is valuable to theorists, however, it is not clear why this should be important to nature. The assumption of low energy supersymmetry restricts the structure of the effective action and permits inferences about the strong coupling and small radius regimes. Plausible assumptions about the distribution of the lagrangian parameters (e.g. uniform distribution of complex parameters in the superpotential) can be checked in the weak coupling region. For example, one can argue that there should be many metastable and stable states even for small radius, and make arguments for the distribution of supersymmetry breaking scales and cosmological constants.<sup>2</sup> But while supersymmetry provides many simplifications and a greater degree of control, one expects that there should be more non-supersymmetric, metastable states, possibly vastly more. There have been some studies of the statistics of non-supersymmetric states, both with spontaneous breaking [4] and explicit breaking [5]. Most of these studies have involved AdS vacua. Attempts to study broader classes of non-supersymmetric vacua include those of Douglas and Denef in the case of IIB orientifolds on Calabi-Yau spaces [7, 8]. Their counting requires approximate supersymmetry. To obtain finite results, a cutoff must be imposed on the scale of supersymmetry breaking. Without such a cutoff, there is no control over the calculations. The vast majority of states are then located at the cutoff scale. Questions such as stability against tunneling are difficult to address for such states. A more ambitious program is that of Silverstein, who argues that there may be various constructions which yield large numbers of non-supersymmetric, de Sitter vacua [6], and that one may have a high degree of control.

One of the most urgent, and potentially accessible, questions in the landscape is the origin of the gauge hierarchy. Is it due to strong dynamics or warping, to supersymmetry, or perhaps just anthropic selection among a vast array of otherwise undistinguished states. In other words, could it be that the explanation of the hierarchy lies not in symmetries or dynamics, but simply in the existence of an overwhelmingly large number states which accidentally have a small scale of weak interactions [9, 10]?

The analysis of Douglas and Denef illustrates why it is hard to settle this question. In a typical, non-supersymmetric state, there will be no small parameters at all, and the crutch of supersymmetry is not available. Unlike the supersymmetric case, there do not seem to be any simple arguments to give a handle on the most rudimentary statistics, much less overall counting.

In the absence of small parameters, one question looms particularly large: stability. If the landscape picture has any validity, the state in which we find ourselves, with small positive cosmological constant, sits in a large sea of states with negative cosmological constant. Many of these are “close by”. Stability of any would-be state requires that the amplitude to decay to any one of these states be very small [11, 12]. Indeed, the very notion of *state* requires this. As we will see, the cutoff of Douglas and Denef can be understood as emerging from the requirement of stability.

The significance of this last point can be understood by supposing that one has a flux landscape of 100 dimensions (i.e. 100 independent fluxes), and typical fluxes are large

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<sup>2</sup>This point was first made to one of the authors (M.D.) several years ago by Shamit Kachru.

(say 10). Then there are of order  $10^{100}$  states. Among these states will be states of small cosmological constant. Any one of these will be surrounded by many states with negative cosmological constant. One might expect, for example, that there are of order  $3^{100}$  within three flux units. In order that the state be stable, it is necessary that the tunneling amplitude to any one of these states (more precisely to the corresponding big crunch) be small. In the absence of a small parameter, one might imagine that there is a probability of order  $1/2$  that the tunneling amplitude be zero to any one state [12]. So the chance that any particular state is stable, absent any symmetries or small parameters, is of order

$$P_{\text{stab}} = \left(\frac{1}{2}\right)^{3^{100}}. \quad (1.1)$$

One might immediately object that there might be qualitative reasons why a particular state does not decay rapidly — or at all — to any of its neighbors. But this is precisely what makes this question important: long-lived states are likely to be special. Optimistically, they might have features related to phenomena we see — or better, might hope to see, in nature. Within the landscape, a number of classes of states have been isolated with distinguishing features. It is natural to ask which of these features might contribute to stability (In what follows, we use “false” vacuum to describe the candidate metastable state; “true” refers to any prospective decay channel):

1. Weak coupling in the “false” vacuum and/or candidate “true” vacua.
2. Large compactification volume in either or both the “false” and “true” vacua.
3. Low energy supersymmetry
4. Light moduli in the “false” vacuum.
5. Warping in the “false” vacuum.

In this note, we investigate these possibilities. To be concrete, we consider mainly Type IIB theories compactified on Calabi-Yau spaces with fluxes and an orientifold projection. In this case, the large number of would-be metastable states is due to a large number of possible flux choices. There is, of course, the risk with such specialization that our results are not sufficiently generic. For example, with our present knowledge of IIB theories, it is difficult to make statements about compactification radii (without supersymmetry), yet as we will see, large radius is a regime (unlike weak coupling) where one might have a realistic hope to find large metastable neighborhoods. The constructions of [6] may yield vast sets of non-supersymmetric, large volume compactifications. We will use these, and AdS models in IIA theory, to give some insight into possible behaviors with volume.

In the end, of the list above, we will argue that only the large volume and supersymmetric states are generically stable. The rest of this paper is organized as follows. We first discuss some general scaling arguments for tunneling amplitudes. These arguments make clear why states with large flux are prone to rapid decay. We review the argument that supersymmetric states are stable. We then consider the (supersymmetric) states discussed

by Giddings, Kachru and Polchinski (GKP) [13]. We verify our scaling arguments for domain wall tensions and cosmological constants. Because of supersymmetry, these states are stable; we will see that this is again consistent with simple scaling arguments.

From these exercises, we confirm that our basic scaling arguments for domain wall tensions and energy splittings are robust. We then suppose that one has found a landscape of non-supersymmetric states, and ask what features might account for stability. We find that while weak coupling, by itself, cannot account for metastability, large volume — more precisely volume scaling suitably with flux,  $N$  — can. Warping, in the sense discussed by GKP seems not to lead to stability. The existence of approximate moduli, by itself, also does not lead to stability. We consider states with a small breaking of supersymmetry (compared to the fundamental scale), and illustrate in simple models why these are typically metastable or completely stable.

While this sort of reasoning can establish classes of states which are metastable, it does not indicate whether one is likely to make transitions *into* a particular state. This is closely related to the questions of measures for eternal inflation which have been widely studied recently. While we currently have little new to add to this discussion, we point out that the landscape is likely to be more complicated than assumed in many simple models of eternal inflation. For example, a typical KKLT vacuum is likely surrounded by many  $AdS$  states, both supersymmetric and not. Whether one can neatly transition into the KKLT minimum seems a serious question. We speculate that states with (discrete) symmetries, though rare, might be attractors in cosmological evolution.

In the conclusions, we indulge in conjecture. Our results, we note, hardly prove that low energy supersymmetry is a feature of the landscape, but they suggest, in ways we explain, that it might be. They suggest, alternatively, what is required to establish the existence of a vast set of non-supersymmetric states in the landscape.

## 2. Scaling arguments for non-supersymmetric states

In this section, we ask how we might expect potentials, domain wall tensions, and tunneling amplitudes to scale in the limit of large fluxes, in non-supersymmetric states. In order that in these hypothetical states there be some validity to a semiclassical analysis in a ten dimensional effective field theory, we will suppose that the compactification volume,  $V$ , is large. We will also assume, when necessary, that couplings are small. We suppose that we have many three-form fluxes ( $b$ ), with typical values of order  $N$ . The potential is quadratic in  $N/V$  (in the Einstein frame, i.e. in four dimensional Planck units).

We are considering a Type IIB landscape, with various RR and NS 3 form fluxes,  $N_i$  and  $K_i$ . We will think of the fluxes as very large,  $N_i \sim N$ , and  $K_i \sim K$ , for some large  $N$  and  $K$ . We wish to compare neighboring states in the landscape, i.e. states in which one flux, say,  $N_i$ , changes by one, or perhaps a few fluxes change by a few. Take the first case. For simplicity, suppose first that  $K \sim N$ . In this case, there is no particular reason for a semiclassical approximation to be valid, but we will use the classical formulas for the potential in order to get some feeling for how amplitudes might scale with  $N$  in a “typical” state. Later, we will adjust the fluxes so that the string coupling is weak.

When all fluxes are comparable, because the potential is homogeneous in  $N$ , the changes in the moduli fields are of order  $1/N$ , and the change in the potential is of order  $N$  for small changes ( $\Delta N \ll N$ ) in flux.

In the decays of interest to us, both the four dimensional fields and the fluxes change, and General Relativity plays an important role. But first it is worthwhile to review some aspects of tunneling in ordinary field theory, without gravity. For our discussion it is important to recognize that even though the barrier may be quite high, if the neighboring well is very deep and the field excursions are not too large, the tunneling amplitude is not necessarily small.

Consider a theory of a scalar field  $\phi$ , in 4 dimensions, with a potential of the form:

$$V(\phi) = \frac{N^2}{V^2} f(\phi). \tag{2.1}$$

Here  $N$  is supposed to be large, as in our problem above, and  $f(\phi)$  is a function with two minima. Let's first ask about the range of validity of the semiclassical analysis in the two would-be minima. Assuming a canonical kinetic term, corrections to the kinetic term, at one loop, behave as  $\frac{N^2}{V^2}$  (the vertices each give a factor of  $\frac{N^2}{V^2}$ , and there is a factor of  $1/m_\phi^2$  from the integral). So the perturbative analysis would seem to be valid if  $V \gg N$ . We will see that stability seems to give a somewhat stronger condition.

Turning to tunneling, ignoring gravity, the bounce action behaves as  $V^2/N^3$ . This follows from simple scaling arguments on the terms in the action. But it can be seen by considering the standard thin wall analysis, which will be valid in the case that the two minima of  $f$  are nearly degenerate, differing by  $\Delta E$ . Then the bubble tension is given by ( $\Delta\phi \sim 1/N$ )

$$T = \int^{\Delta\phi} d\phi \sqrt{2V(\phi)} \sim \frac{1}{V}, \tag{2.2}$$

so the standard thin wall analysis [11] gives

$$S_b = C \frac{T^4}{\Delta E^3} = AV^2/N^3, \tag{2.3}$$

where  $C$  and  $A$  are numerical constants. So unless the volume is large (of order  $N^{3/2}$  in fundamental units), the bounce action is small and the tunneling amplitude is of order one. This same scaling can readily be shown to hold for more general functions  $f$ .

Typically, gravitational corrections will be large. From the Coleman-DeLuccia analysis, in the case of tunneling, say, from flat or AdS space to AdS space, one learns that gravitational corrections are important when the radius of the would-be bubble is comparable to the AdS radius. In our example above, the radius of the bubble wall is:

$$R_b = \frac{T}{\Delta E} \sim V/N \tag{2.4}$$

while the AdS radius is of order  $V/N$ .

On the other hand, for the case of interest to us in the landscape, the initial state, by assumption, has zero or nearly zero cosmological constant. So,

$$R_{\text{AdS}} \sim \frac{V}{\sqrt{N}} \tag{2.5}$$

and one does not expect gravitational corrections to be important. More generally, one does not expect a suppression of decays from states with large positive to negative cosmological constant.

Of course, by definition, if the bounce action is not large, the semiclassical calculation is not reliable. We take this result as evidence that suppressing tunneling requires  $N^3 \ll V^2$ . Note that this constraint is more severe than the naive perturbative condition,  $V \gg N$ .

When we consider flux vacua, we will be interested in the Coleman or Coleman-DeLuccia analysis in cases where the bubble can be thought of as a brane ( $M5$  or  $D5$ ) wrapped on some internal manifold. Such branes can be thought of as thin-walled bubbles. To make the comparison, it is useful to reformulate the conventional field theory analysis in terms of a collective coordinate. If the bounce is described by a function  $\phi_{cl}(r - R)$  (spherically symmetric), we can introduce a collective coordinate,  $R(t)$ , and obtain an action for  $R$  by writing:

$$\phi(r) = \phi_{cl}(r - R(t)) : \tag{2.6}$$

$$S = \int dt \left( TR^2 \dot{R}^2 - V(R) \right) \tag{2.7}$$

with

$$V(R) = TR^2 - \Delta ER^3. \tag{2.8}$$

The bounce action follows from an ordinary WKB calculation with this action.<sup>3</sup> For the case of wrapped branes, this is the structure of the appropriate Born-Infeld action.

The estimates in this section suggest that the tunneling amplitude quickly becomes of order one as  $N$  increases, if  $V$  and the couplings are fixed. But one might worry that this field theory analysis is not applicable to the case of interest, where transitions are accompanied by changes of flux and emission of extended objects like branes. In the next section, we will see that precisely these scalings of tensions and cosmological constants occur in well-studied string theory examples.

### 3. A prototype: GKP

A useful model for stabilized moduli and domain wall tensions is provided by the work of Giddings, Kachru and Polchinski. This is a IIB orientifold model, with compactification on a Calabi-Yau space at a point in the (approximate) moduli space near a conifold singularity.

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<sup>3</sup>In terms of our earlier remark about quantum tunneling, it is the unusual form of the kinetic term for  $R$  and the pressure term which account for the enhanced tunneling rate.

In this model, the moduli are  $\tau$ ,  $\rho$  and  $z, z_i$ .  $\tau$  is the IIB string coupling;  $\rho$  describes the overall size of the compact manifold,

$$\rho = R^4 M_{10}^4 \tag{3.1}$$

(where  $M_{10}$  is the ten-dimensional Planck scale), and  $z$  is a complex structure modulus which describes the deformation of the conifold. The  $z_i$  represent other complex structure moduli.  $\tau$ ,  $z_i$  and  $z$  are fixed by fluxes;  $\rho$  is undetermined in a semiclassical treatment (the model has a no-scale structure). It will be useful to consider an effective field theory without  $\rho$ , as well, in which supersymmetry is unbroken.

For large  $\rho$ , the theory turns out to be approximately supersymmetric. Following GKP, we will consider light fields  $z$ ,  $\tau$  and  $\rho$ . Their dynamics can be described by a superpotential and Kahler potential. The superpotential takes the form:

$$W = M\mathcal{G}(z) - K\tau z - K'\tau h(z) \tag{3.2}$$

where  $M$  is the RR three-form flux along the  $A$  cycle associated with the conifold;  $K$  is the NS-NS three form flux along the corresponding  $B$  cycle; and  $K'$  is the flux along some other cycle,  $B'$ . The function  $\mathcal{G}$  has the form:

$$\mathcal{G}(z) = \frac{z}{2\pi i} \ln(z) + \mathcal{G}(0) + f(z) \tag{3.3}$$

where  $f(z)$  is a holomorphic function of  $z$  which vanishes as  $z \rightarrow 0$ . The Kahler potential

$$K = -3\ln(\rho - \rho^*) - \ln(\tau - \tau^*) + f(z, z^*) \tag{3.4}$$

where  $f(z, z^*)$  is a finite function of  $z$  which tends to a constant as  $z \rightarrow 0$ .

For suitable choices of flux, the equations for supersymmetric stationary points of  $z$  and  $\tau$  have solutions with  $z$  small and  $\tau$  large:

$$z = \exp\left(\frac{2\pi K \text{Im } \tau}{M}\right) \quad \text{Im}\tau = -\frac{M\mathcal{G}(0)}{K'h(0)}. \tag{3.5}$$

As noted above, at these points,  $\rho$  is undetermined and the potential, classically, vanishes. For large  $\rho$ , the gravitino mass is small and one can argue that the computation is self-consistent. The masses of the lightest Kaluza-Klein modes are of order

$$m_{\text{KK}}^2 \sim \frac{1}{\rho^2}, \tag{3.6}$$

where this, which is expressed in four dimensional Planck units, applies in the limit of large  $\rho$  and moderate  $z$ .

The masses of the gravitino,  $m_{3/2}$ , is of order:

$$m_{3/2}^2 \sim |M\mathcal{G}(0)|^2 \rho^{-3} g_s, \tag{3.7}$$

justifying the use of a the supersymmetric lagrangian, for large  $\rho$  (and/or small  $z$ ). The masses of  $z$  and  $\tau$  are of order:

$$m_\tau^2 \sim |M\mathcal{G}(0)|^2 \frac{g_s}{\rho^3} \quad m_z^2 \sim M^2 \frac{g_s}{\rho^3 z^2}. \tag{3.8}$$



So the inclusion of  $\tau$  in the low energy effective lagrangian is sensible; for fixed  $\rho$ , however,  $z$  becomes massive as  $z \rightarrow 0$ , and a more careful analysis seems required. We will proceed without worrying about this potential subtlety, as we will be generally interested in rough scalings of tunneling amplitudes, in any case.

### 3.1 Tensions and cosmological constants

The vacua in this leading approximation are all degenerate (zero cosmological constant), so this model is not useful for discussing tunneling amplitudes. As a toy model, we can consider a theory without  $\rho$ , with minima which are supersymmetric and AdS. However, because of supersymmetry, there is still no tunneling between the vacua. The model is useful, however, for studying how domain wall tensions and vacuum energies (cosmological constants) scale with flux, and also how warping effects these quantities. Without  $\rho$  (or with  $\rho$  fixed), the domain wall tension between vacua of different flux is [14]:

$$T = 2\Delta|e^{K/2}W|. \tag{3.9}$$

We can consider various types of transitions. For transitions with  $\Delta M = \pm 1$ , the tension is given by

$$\begin{aligned} T &\approx 2 \left| \mathcal{G}(0)e^{K/2} + e^{K/2} \left( K'h(0) - \frac{1}{2\tau} [M\mathcal{G}(0) + \tau K'(0)] \right) \Delta\tau \right| \\ &\sim 2|\mathcal{G}(0)|\rho^{-3/2}g_s^{1/2}. \end{aligned} \tag{3.10}$$

Where we used that at the minima  $\tau = -\bar{\tau}$  and neglected contributions of order  $z$ . This is in accord with the general scaling arguments of section 2; the tension is of order one, if all fluxes are scaled uniformly. In addition, in the theory absent  $\rho$ , the change in the cosmological constant is of order  $M$ , again in accord with our general scaling arguments.

For those with  $\Delta K' = \pm 1$ , the tension is of order:

$$T \sim M/K'\rho^{-3/2}g_s^{1/2}. \tag{3.11}$$

Again, if all scales become uniformly large, this is in accord with our earlier arguments. Interestingly, however, for small string coupling, this is enhanced relative to the  $\Delta M = 1$  transitions. The change in the cosmological constant is of order

$$M^2/K'\rho^{-3}g_s. \tag{3.12}$$

Finally, changes in  $K$  are associated with domain walls with tension suppressed by  $z$ , and with changes in cosmological constant similarly suppressed.

### 3.2 GKP as a prototype for non-supersymmetric scalings

Because of the supersymmetry of the GKP solution, without  $\rho$ , or the degeneracy with  $\rho$ , semiclassically there is no tunneling. But we have seen that the domain wall tension and energy splittings behave as expected for theories without supersymmetry, so we can use the GKP solution as a model for transitions among non-supersymmetric states. We would then

expect that tunneling amplitudes would behave roughly as  $e^{-S_b}$ , where  $S_b \sim T^4/\Delta E^3$ . So, for example, for the for transitions with  $\Delta M = 1$ , we would have, for the bounce action:

$$S_b = \frac{e^{-K}}{M^3} \sim \frac{V^2 \tau}{M^3} \sim \frac{V^2}{M^2 K'} \tag{3.13}$$

Here we have used the flux dependence of  $\tau$ , eq. (3.5) This flux and volume behavior is as we anticipated in section 2, in the sense that it involves three powers of flux in the denominator, though one of these factors is the small (NS-NS) flux.

In any case, this model is compatible with our naive estimates: obtaining a large set of metastable non-supersymmetric compactifications would seem to require that the volume scale as a power of the flux. Following our discussion in section 2, we would expect that, while in general, gravitational corrections are important and might suppress the decay amplitude in some cases, this is not the case if the initial state has small cosmological constant.

This has a close parallel to the field theory discussion along the lines of [11, 12]. For the case of a change of one unit of RR flux, the expanding bubble can be thought of as a wrapped  $D5$  brane. Three of the directions along the brane are wrapped on a three cycle; the remaining two dimensions correspond to the bubble wall. Of the four collective coordinates of the brane, three are located at a point on the internal manifold; the remainder is the coordinate,  $R(t)$ , describing the wall surface. The tension of the  $D5$  brane is just the tension we identified before:

$$\begin{aligned} T &= \frac{1}{g_s} M_s^6 (R^3 M_{10}^3) M_{10}^{-3} \\ &= \frac{1}{g_s} \rho^{-3/2} M_p^3 \end{aligned} \tag{3.14}$$

which is what we found from the heuristic field-theoretic argument.

We need, also, to worry about conservation of  $D3$  brane charge. In general, transitions involving changes in flux will involve emission of  $D3$  branes, as well. We won't consider this problem in detail. However, for special cases (those for which  $\int H \wedge F$  vanishes), there need be no  $D3$  brane emission. In cases where there is brane emission, the energetics are the same as suggested by the field theory arguments. For a change in  $K$  of order one, one needs a change in the  $D3$  brane density of order  $M$ . Wrapped  $D3$  branes have tension (mass) of order  $1/g_s M_p$  (independent of  $\rho$ ), so the energy density is of the same order as the change in energy due to the change in flux. So we expect that our estimates of tunneling rates above are still correct.

#### 4. Tunneling from approximately supersymmetric vacua

From the point of view of stability, supersymmetric vacua are special. According to quite general arguments, they are stable. This is perhaps surprising, since in supergravity it is perfectly possible for a lagrangian to exhibit a supersymmetric state (say with vanishing cosmological constant) and a non-supersymmetric state with large, negative cosmological constant. In such a case, one does not expect, for example, a BPS domain wall.

The essential point was made by Deser and Teitelboim [15], Witten [17], Hull [16] and others long ago. They noted that in a classical supergravity theory in an asymptotically flat space, one can define not only a total energy and momentum, but also global supercharges. These obey the standard supersymmetry algebra, so, just as is familiar in global supersymmetry, the energy of any configuration can be shown to be greater than or equal to zero. (This is a special case of the positive energy theorem.)

The stability of exactly supersymmetric states may or may not be of interest, but clearly the stability of *approximately* supersymmetric states is of great potential importance. Suppose the scale of supersymmetric breaking,  $F$ , is small compared to the Planck scale and other possible scales of interest (string scale, compactification scale). Then the decay probability behaves as

$$\Gamma \sim e^{-1/|F|^2} \tag{4.1}$$

or vanishes. It is only non-vanishing if susy breaking in the AdS state is comparable or smaller than that in the approximately flat space state.

#### 4.1 Models

These features of decays of (nearly) supersymmetric states can be illustrated with simple models. Consider, first, a theory of a single scalar field,  $\phi$ , with superpotential:

$$W = \frac{1}{2}M\phi^2 - \frac{1}{3}\gamma\phi^3. \tag{4.2}$$

Before coupling to gravity, this theory has supersymmetric minima at

$$\phi_0 = 0; \quad \phi_0 = \frac{M}{\gamma}. \tag{4.3}$$

These can be joined by a domain wall, with tension:

$$T = 2\Delta W. \tag{4.4}$$

Now couple the system to (super)gravity, with  $M \ll M_p$ . In this case, the domain wall tension is approximately unchanged, but there is a splitting between the states,

$$\Delta E = 3 \frac{|\Delta W|^2}{M_p^2} \tag{4.5}$$

$\Delta E$  is small compared to the scales in the superpotential, so a thin wall approximation is appropriate. The bubble radius is, again,

$$R_b = \frac{3T}{\Delta E} \tag{4.6}$$

As explained by Coleman and DeLuccia, the decay amplitude vanishes if

$$\frac{R_b}{\Lambda} = 2 \tag{4.7}$$

and precisely this condition is satisfied in this model.

Now let's add supersymmetry breaking to the mix. This can be done by adding an additional chiral field,  $Z$ , and taking for the superpotential:

$$W = \frac{M}{2}\phi^2 - \frac{\gamma}{3}\phi^3 + Z\mu^2 + W_0. \tag{4.8}$$

The  $Z$  field can be stabilized in both vacua by adding a term  $Z^\dagger Z Z^\dagger Z$  to the Kahler potential.  $W_0$  is chosen so that the  $\phi = 0$  state has zero cosmological constant:

$$3|W_0|^2 = |\mu^4|. \tag{4.9}$$

Note that the phase of  $W_0$  is not fixed by this condition.

For this system, the bubble wall tension is approximately as it was in the previous case, but the energy shift is different. Calling the original shift  $\Delta E_0$ , the shift is larger or smaller by an amount of order  $FM^3/(\gamma^2 M_p)$ , depending on the sign of  $W_0$ . In the latter case, the amplitude vanishes; in the former, it is of order

$$\Gamma \approx e^{-6\pi^2 M_p^4 / |F|^2}. \tag{4.10}$$

Note that even for moderately small  $F$  (in whatever are the appropriate units) and in the presence of an exponentially large number of decay channels, the decay rate is extremely small. We might expect that  $|F| \sim 10^{-3} M_p^2$  would more than adequately suppress the decay rate. The implications of this observation for a possible *prediction* of low energy supersymmetry will be discussed in the conclusions.

### 5. Large volume, weak coupling, light moduli and warping

From our studies of the GKP model, we can already see that weak string coupling, by itself, does not insure stability. However, a combination of weak coupling and large volume does. What is required is that there be a large number of states whose volume scales with a power (3/2) of the flux. One suspects, more generally, that whatever is the parameter(s) which account for the exponentially large number of states, the volume must scale with a power of this parameter.

In the IIB constructions which have been studied, the fixing of the volume is not well understood (except in special cases which have exact or approximate low energy supersymmetry). In IIA theories, however, candidate vacua have been identified with all moduli fixed [4, 5]. In these cases, one has an infinite sequence of AdS vacua, with or without supersymmetry, with progressively larger volume and smaller cosmological constant. We can again take these as a model, supposing that there exists a set of dS vacua with similar scalings, and ask how the tunneling amplitudes would behave.

Without reviewing the IIA models in detail, we note that the important large number in these constructions is the four-form flux, which we will refer to generically as  $N$ . As in the IIA case, the action scales as  $N^2$ . At the minimum, one finds that the volume,  $v$ , and the dilaton,  $e^\phi$ , scale as

$$v \sim N^{3/2} \quad e^\phi \sim N^{-3/4} \tag{5.1}$$

and the cosmological constant behaves as  $N^{-9/2}$ . As a result, the tension of the bubble walls and the energy splittings between states behave as

$$T \sim N^{-13/4} \quad \Delta E \sim N^{-11/2} \quad (5.2)$$

giving a result for the bounce action which grows rapidly with  $N$ :

$$S_b \sim N^{3.5}. \quad (5.3)$$

Another model for scaling with volume is provided by the proposal of Silverstein and Saltzman to compactify string theory on products of Riemann surfaces [6]. Again, without reviewing the details of the model, there are various numbers, such as 5-form flux,  $q_5$ , which can become large, accounting for the large number of states, many of which are believed to be de Sitter. The volume, in this case, scales as  $q_5^3$ , while the vacuum energy scales as  $V^{-4/3}$ . So again the bounce action grows as a power of the large parameter.

Both of these models suggest that if there do exist large sets of de Sitter vacua with growing volume and decreasing cosmological constant, they are likely to be highly metastable.

Warping, on the other hand, does not seem, by itself, to lead to suppression of tunneling rates, as we see from the GKP model. One might have expected this in any situation where there are large numbers of fluxes. The GKP construction suggests that warping is obtained by tuning some set of fluxes on cycles associated with the warp region; changing far away fluxes, then, may be possible without spoiling this feature. Formulas (3.10)–(3.12) exhibit no singular dependence on  $z$ .

## 6. Tunneling from states with light moduli

Supersymmetric vacua with moduli are very familiar. Much of the recent focus on flux vacua is motivated by the observation that these have few or no light moduli. Still, we might speculate that there exist classes of non-supersymmetric vacua with comparatively light pseudomoduli. Those with small cosmological constant would likely have neighbors with negative cosmological constant, and no moduli at all.

One can ask whether the presence of light moduli would somehow suppress tunneling. In the standard treatment (we will ignore the effects of gravity in this section) one needs to study an analog problem, the motion of a particle in a potential, with boundary conditions that in the far future, the system settle into the false minimum of the potential. One might hope that the light field would only slowly settle into its minimum, giving rise to a large bounce action. This turns out not to be the case, in general.

In the transition, one expects a significant rearrangement of the degrees of freedom. The final state in the transition likely will have no light fields, or at least different numbers of them. High energy string states in one vacuum might be relatively light states in another. To develop some intuition, we consider a field theory with two fields,  $X$  and  $\phi$ .  $\phi$  is light in the “false” vacuum but heavy in the “true” vacuum;  $X$  is heavy in both. For the potential we take:

$$V = \frac{1}{2}\mu^2\phi^2 + \frac{M^2}{2}X^2 - \frac{1}{4}X^4 + \frac{\Gamma}{6}X^6 + \epsilon X^2(\phi - \phi_0)^2. \quad (6.1)$$

The idea here is that  $\mu$  is extremely small compared to  $M, \Gamma$ , which define mass scales of the same order. To permit simple calculations, we can take  $\epsilon$  to be a small number.  $\Gamma$  is chosen so the vacuum with  $X \neq 0$  has lower energy than the  $X = 0$  vacuum. Again, to allow simple approximations, we can tune  $\Gamma$  so that the energy difference between the  $X = 0$  vacuum and the  $X \neq 0$  vacuum is, say, of order  $\epsilon M^4$ .

For small  $\epsilon$ , we can consider first the dynamics of  $X$  by itself. We will also ignore gravity at first. This is then a standard thin wall tunneling problem. The bounce has a size of order  $r_0 = t_0 = 1/(\epsilon M)$ , where  $r_0$  is meant to denote the bubble radius and  $t_0$  denotes the typical time in the analog particle problem. In the particle analogy with inverted potential,  $X$  starts extremely close to the top of the hill (the true vacuum),

$$X - X_0 \sim M e^{-Mt_0}. \tag{6.2}$$

It then rolls quickly to the false vacuum. As it approaches the false vacuum ( $X = 0$ ), it behaves, again, as

$$X \sim M e^{-M(t-t_0)}. \tag{6.3}$$

For sufficiently large time ( $M(t-t_0) \sim \ln(\frac{\epsilon M^2}{\mu^2})$ ),  $\epsilon X^2 \approx \mu^2$ . Until that time, the minimum of the  $\phi$  potential lies at  $\phi_0$ . In the inverted problem,  $\phi_0$  then rolls away from that point (towards the origin).

Note, however, that this time is much smaller than  $\mu^{-1}$  (it is also smaller than  $r_0$ ). So  $\phi$  satisfies the equation:

$$\ddot{\phi} + \frac{3}{t} \dot{\phi} = 0. \tag{6.4}$$

This has solutions  $\phi = 1/t^2 + \text{constant}$ . By tuning the initial conditions, one can arrange that  $\phi < \mu$  in a time much less than  $\mu^{-1}$ ; for such motions, there is no enhancement of the bounce action.

## 7. Implications and speculations

From this survey, we have concluded that generic, metastable states, are likely to satisfy special conditions. We have identified two possibilities:

1. (Approximate) Supersymmetry at scales well below the fundamental scale.
2. Compactification radii much larger than the string scale

and ruled out several others.

The strongest indication for the existence of large numbers of large volume, dS states comes from the work of [6], but the work of [7, 4] is also suggestive. For many of these constructions, there are good reasons for skepticism (see, e.g., [19], though it should be noted that there are also reasons to be skeptical of supersymmetric constructions). Even if such states exist, it is conceivable that many are in some sense uninteresting, since coupling constants become small in the large volume limit. If, in the end, the explanation of hierarchy

is the existence of a vast array of non-supersymmetric states (with large volume or some other feature which accounts for metastability), the question will be: are these states distinguished in some other way? Otherwise, observations of phenomena at accelerator energies will provide us with no information about physics at extremely high energies.

One might speculate that there do not exist such large sequences of large volume compactifications, no some other generic vast class of non-supersymmetric metastable dS states. Stability might favor low energy supersymmetry. It is then interesting to ask whether among these states, there might be further selection effects.

### 7.1 R symmetric points as cosmological attractors

Having established that some set of states are metastable, it is natural to ask whether the universe might find its way into them. To address this issue, the first task is to survey the neighborhood of some particular state (or class of states) of interest. Let's begin with the KKLT vacua. It seems likely that these are surrounded by AdS vacua, largely non-supersymmetric. From our arguments above, the KKLT vacua are likely stable against decays to these states. Because of these many AdS states, transitions *into* the KKLT vacua might be difficult.

A potentially interesting set of states are those which exhibit discrete R symmetries. Such states are likely rare in the landscape [18], but their neighborhoods may be more interesting. In general, one finds such states by setting to zero all fluxes which transform non-trivially under the candidate symmetry. Typically, this is a substantial fraction of the fluxes — 2/3 or more.

Arguably this is what one expects for symmetric states: they are special and thus rare. But the idea that cosmological considerations (e.g. high temperatures) can favor symmetric states is familiar. For the flux lattice, things cannot be so simple; we are concerned about discrete transitions. But it seems possible that such symmetric states might be cosmological attractors. Consider the neighborhood of the  $R$  symmetric states, i.e. states for which the symmetry preserving fluxes,  $N_i$ , are large, while those which break the symmetry are much smaller, say  $n_\alpha$ . Similarly, there are fields,  $Z_i$ , which transform like  $W$  at the  $R$  symmetric point, and fields,  $\phi_a$ , which transform differently. At the  $R$  symmetric point, there is a superpotential:

$$W = Z_i f^i(\phi_a, Z_i) \tag{7.1}$$

The condition for unbroken supersymmetry is:

$$Z_i = 0; \quad f_i(\phi_a, 0) = 0. \tag{7.2}$$

Provided there are more  $\phi_a$  type fields than  $Z_i$  fields, these equations, generically, possess a moduli space of solutions.

Now treat  $N_i$  and  $n_\alpha$  as sufficiently large that they can be thought of as continuous, with  $|N_i| \gg |n_\alpha|$ , for all  $i$  and  $\alpha$ . As we turn on small  $n_\alpha$  we can study an effective lagrangian for the light moduli of the symmetric vacuum. There is no reason to think that the structure of stationary points of the effective lagrangian for this potential is different

than the general (non-R) case. So there will typically be solutions with broken supersymmetry, and positive or negative cosmological constant [7]. Introducing polar coordinates,  $\vec{n} = (n, \theta_1, \theta_2, \dots)$ . The cosmological constant, as a function of  $\vec{n}$ , then has the structure:

$$\Lambda(\vec{n}) = n^2 v(\theta_i). \tag{7.3}$$

It is then plausible that there are finite elements of “solid angle” with either sign of the cosmological constant, and in particular finite regions with positive cosmological constant and energy tending to zero as  $n \rightarrow 0$ . For these regions, the symmetric state may function as a *cosmological attractor*. Since most “jumps” will be very rapid, it is perhaps appropriate to think of  $\vec{n}$  as a collective coordinate, and the motion in this space as reasonably smooth. Note that in the last few transitions, the continuous approximation for  $n$  will break down, and there may be AdS states close to the symmetric point, which may have catastrophic consequences for a particular decay chain. But it is at least plausible that finite domains of the landscape are attracted to R symmetric vacua.

## 7.2 Implications of stability

Of cosmological issues within the landscape, metastability is the simplest criterion which states must satisfy. Of generic features of landscape states, we have seen that only large volume, or approximate supersymmetry, seem to result in some degree of metastability. It is conceivable that we could settle the question of whether vast arrays of large volume, non-supersymmetric states exist in an underlying theory of gravity. As we have noted, if they do, and they are otherwise undistinguished, it is unclear how one might imagine developing a string phenomenology. Not only would we fail to make predictions, e.g. for LHC physics, but we would not know how to interpret LHC outcomes. If not, however, low energy supersymmetry would seem a prediction of string theory.

We should stress, of course, that stability, by itself, does not imply *weak scale* supersymmetry. A quite high scale of supersymmetry breaking will insure an adequate level of stability. However, within the supersymmetric branches of the landscape, the scale of supersymmetry breaking is likely flat on a log scale [3]. So, just as expected from conventional naturalness arguments, light Higgs should arise with low supersymmetry breaking scale.

We have gone further, examining the neighborhood of the KKLT vacua, as well as sets of vacua with approximate  $R$  symmetries, suggesting reasons why cosmology might favor the later. These remarks are on far shakier ground, but are worthy of further study.

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